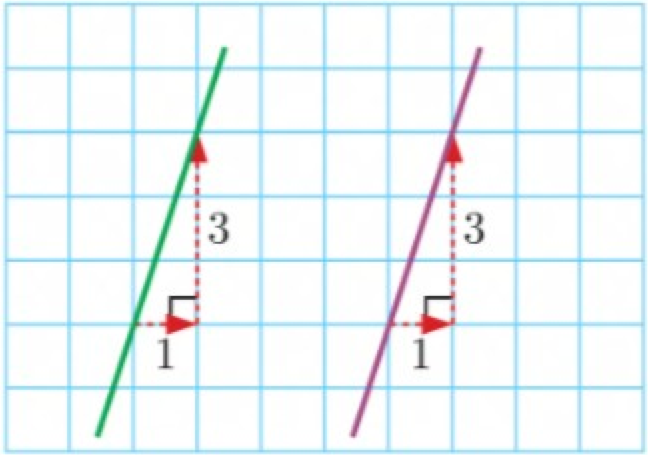
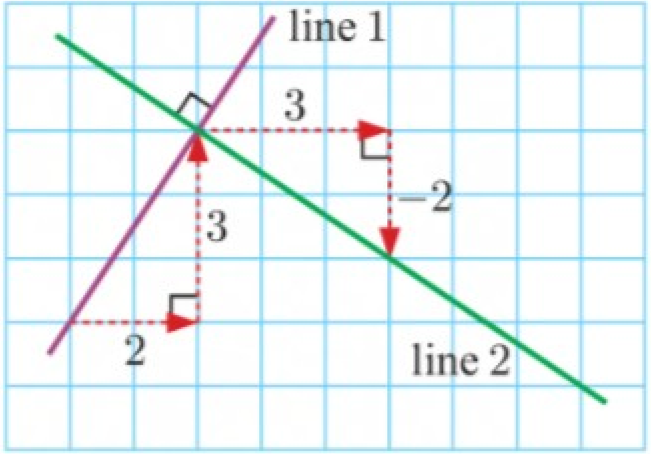
PARALLEL AND PERPENDICULAR LINES

PARALLEL LINES



The given lines are parallel, and both of them have a gradient of 3.

- If two lines are **parallel**, then they have **equal gradient**.
- If two lines have **equal gradient**, then they are **parallel**.



Line 1 and line 2 are perpendicular.

Line 1 has gradient $\frac{3}{2}$.

Line 2 has gradient $\frac{-2}{3} = -\frac{2}{3}$.

We see that the gradients are *negative reciprocals* of each other, and their product is $\frac{3}{2} \times -\frac{2}{3} = -1$.

For lines which are not horizontal or vertical:

- if the lines are **perpendicular**, then their gradients are **negative reciprocals**
- if the gradients are **negative reciprocals**, then the lines are **perpendicular**.

DEMO

Example 11 **Self Tutor**

Find the gradient of all lines perpendicular to a line with a gradient of:

a $\frac{2}{7}$ **b** -5

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Find the gradient of all lines perpendicular to a line with a gradient of:

a $\frac{2}{7}$ **b** -5

a The negative reciprocal of $\frac{2}{7}$ is $-\frac{7}{2}$.

$\therefore$ the gradient of any perpendicular line is $-\frac{7}{2}$.

b The negative reciprocal of $-5 = \frac{-5}{1}$ is $\frac{1}{5}$.

$\therefore$ the gradient of any perpendicular line is $\frac{1}{5}$.

The negative reciprocal of $\frac{a}{b}$ is $-\frac{b}{a}$.

1 Find the gradient of all lines perpendicular to a line with a gradient of:

a $\frac{1}{2}$
b $\frac{2}{5}$
c 3
d 7
e $-\frac{2}{5}$
f $-\frac{7}{2}$
g $-1\frac{1}{3}$
h -1

2 The gradients of two lines are listed below. Which of the line pairs are perpendicular?

a $\frac{1}{3}, 3$
b $5, -5$
c $\frac{3}{7}, -2\frac{1}{3}$
d $4, -\frac{1}{4}$
e $6, -\frac{5}{6}$
f $\frac{2}{3}, -\frac{3}{2}$
g $\dfrac{p}{q}, \dfrac{q}{p}$
h $\dfrac{a}{b}, -\dfrac{b}{a}$

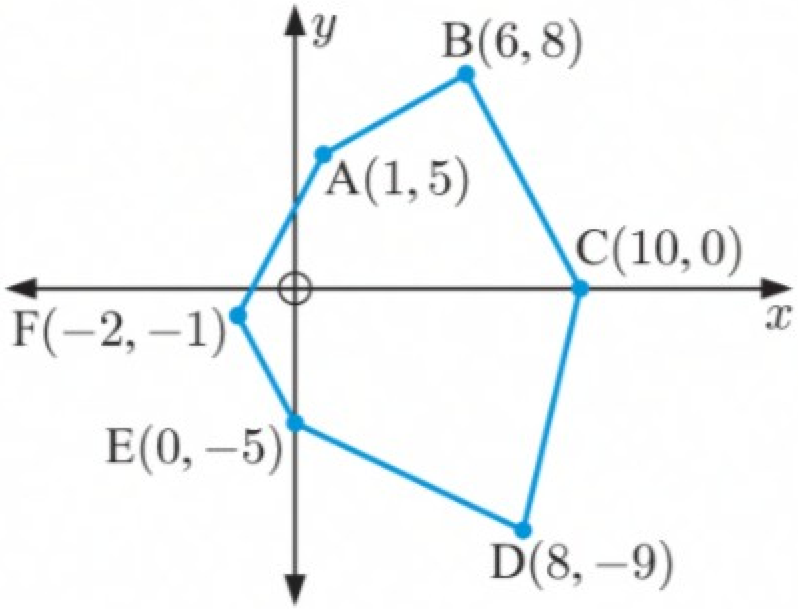
3 Consider the hexagon alongside.

a Calculate the gradient of each side of the hexagon.

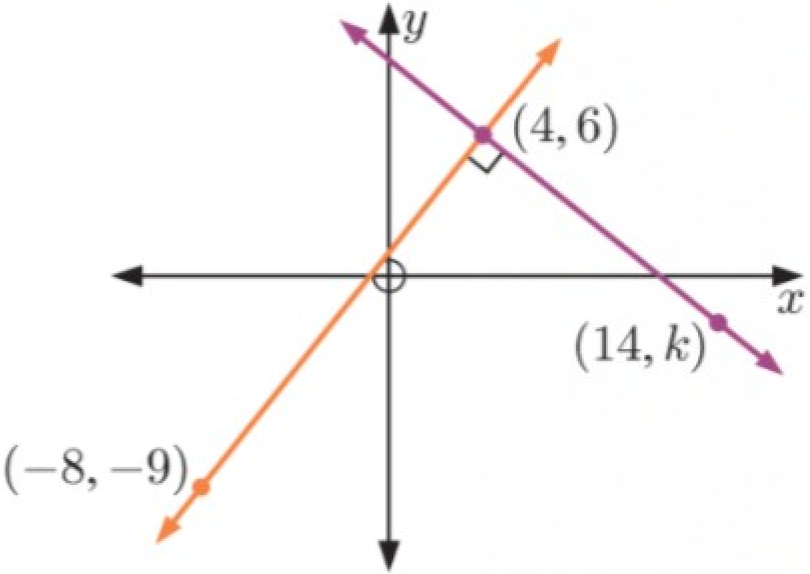
b Which sides are:

i parallel

ii perpendicular?



4 Find the value of k:



5 Consider the points A(1, 4), B(−1, 0), C(6, 3), and D(t, −1). Find t if:

a [AB] is parallel to [CD]

b [AC] is parallel to [DB]

c [AB] is perpendicular to [CD]

d [AD] is perpendicular to [BC].

6 Consider the points P(1, 5), Q(5, 7), and R(3, 1).

a Show that triangle PQR is isosceles.

b Find the midpoint M of [QR].

c Use gradients to verify that [PM] is perpendicular to [QR].

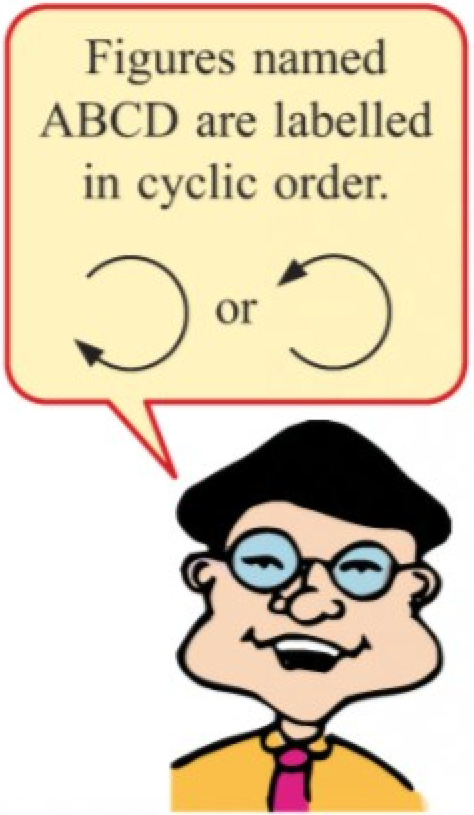
d Draw a sketch to illustrate what you have found.

7 For the points $A(-1, 1)$, $B(1, 5)$, and $C(5, 1)$, M is the midpoint of [AB], and N is the midpoint of [BC].

- **a** Show that [MN] is parallel to [AC].
- **b** Show that [MN] is half the length of [AC].

8 Consider the points $A(1, 3)$, $B(6, 3)$, $C(3, -1)$, and $D(-2, -1)$.

- **a** Use the distance formula to show that ABCD is a rhombus.
- **b** Find the midpoints of [AC] and [BD].
- **c** Show that [AC] and [BD] are perpendicular.
- **d** Draw a sketch to illustrate your findings.


Figures named ABCD are labelled in cyclic order.
or

9 The sketch of quadrilateral ABCD is not drawn to scale. P, Q, R, and S are the midpoints of [AB], [BC], [CD], and [DA] respectively.

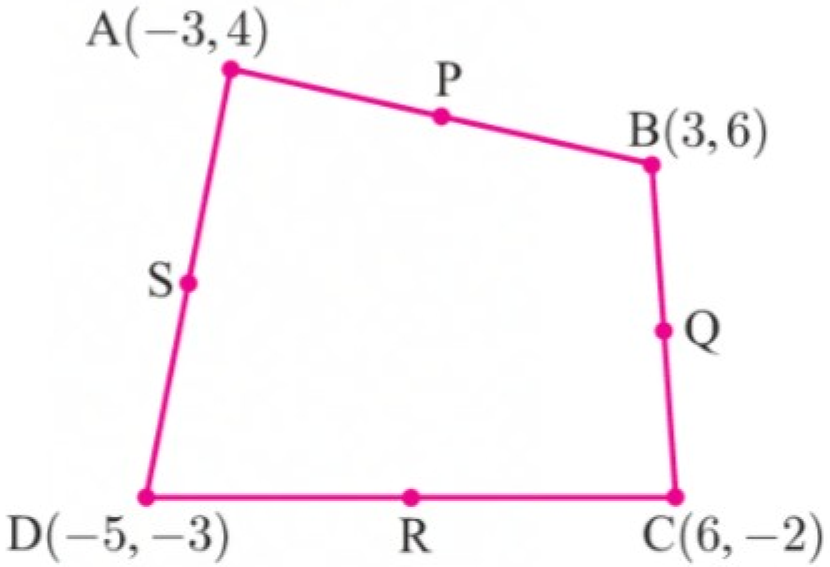
a Find the coordinates of:

i P **ii** Q **iii** R **iv** S.

b Find the gradient of:

i [PQ] **ii** [QR] **iii** [RS] **iv** [SP].

c What can be deduced about quadrilateral PQRS?



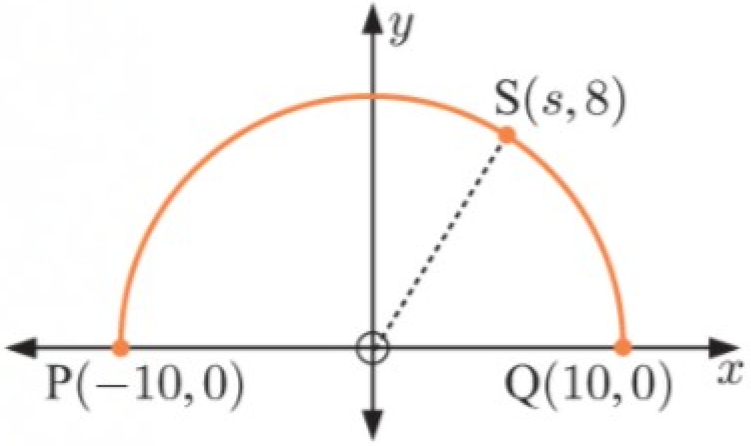
10 $S(s, 8)$ lies on a semi-circle as shown.

a Find s.

b Find the gradient of:

i [PS] **ii** [SQ].

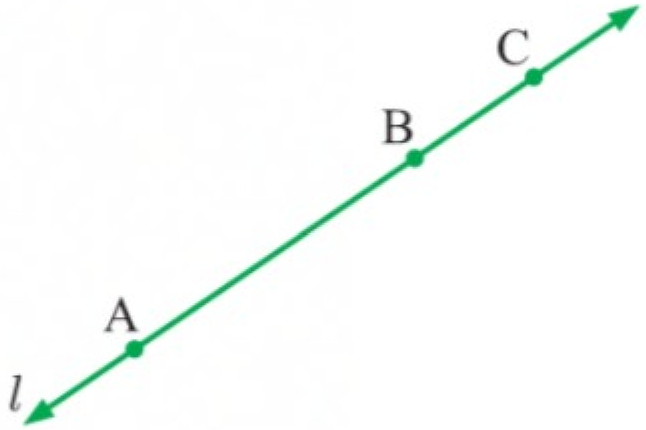
c Hence show that angle PSQ is a right angle.



Three or more points are **collinear** if they lie on the same straight line.

nsider the three collinear points A, B, and C, which all lie on line l.

gradient of [AB] $=$ gradient of [BC] $=$ gradient of l



Three points A, B, and C are **collinear** if gradient of [AB] $=$ gradient of [BC].

Example 12

Self Tutor

Show that the points $A(1, -1)$, $B(6, 9)$, and $C(3, 3)$ are collinear.

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Self Tutor

Show that the points $A(1, -1)$, $B(6, 9)$, and $C(3, 3)$ are collinear.

Gradient of [AB] $= \dfrac{9 - -1}{6 - 1} = \frac{10}{5} = 2.$ Gradient of [BC] $= \dfrac{3 - 9}{3 - 6} = \frac{-6}{-3} = 2.$

$\therefore$ [AB] is parallel to [BC], and point B is common to both line segments.

$\therefore$ A, B, and C are collinear.

EXERCISE

1 Determine whether the following sets of points are collinear:

a A$(1, 2)$, B$(4, 6)$, and C$(-4, -4)$

b P$(-6, -6)$, Q$(-1, 0)$, and R$(4, 6)$

c R$(5, 2)$, S$(-6, 5)$, and T$(0, -4)$

d A$(0, -2)$, B$(-1, -5)$, and C$(3, 7)$.

2 Find c given that these three points are collinear:

a A$(-4, -2)$, B$(0, 2)$, and C$(c, 5)$

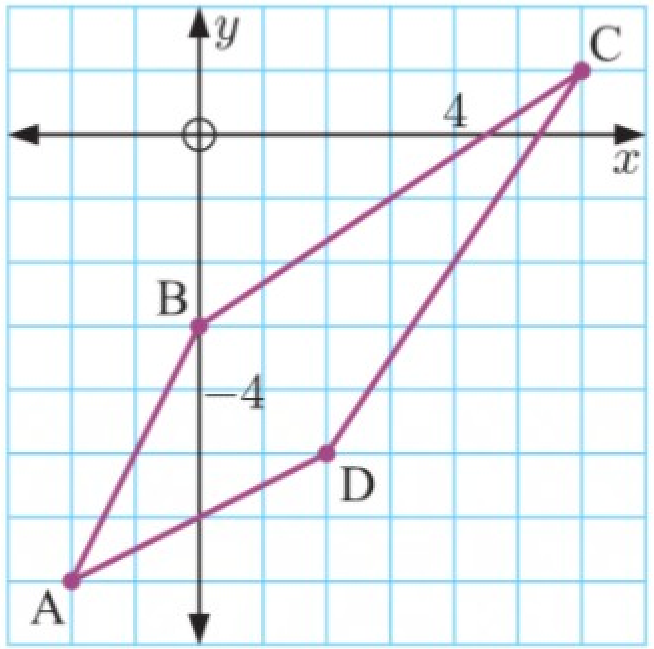
b P$(3, -2)$, Q$(4, c)$, and R$(-1, 10)$.

3 The points A$(-2, -7)$, B$(0, -3)$, C$(6, 1)$, and D$(2, -5)$ form a kite.

a Find the midpoint M of [BD].

b Show that A, M, and C are collinear.

c Show that [AC] is perpendicular to [BD].



EXERCISE 6D.1

1 **a** -2 **b** $-\frac{5}{2}$ **c** $-\frac{1}{3}$ **d** $-\frac{1}{7}$ **e** $\frac{5}{2}$ **f** $\frac{2}{7}$
g $\frac{3}{4}$ **h** 1

2 The line pairs in **c**, **d**, **f**, and **h** are perpendicular.

3 **a** [AB]: $\frac{3}{5}$, [BC]: -2, [CD]: $\frac{9}{2}$, [DE]: $-\frac{1}{2}$, [EF]: -2, [FA]: 2

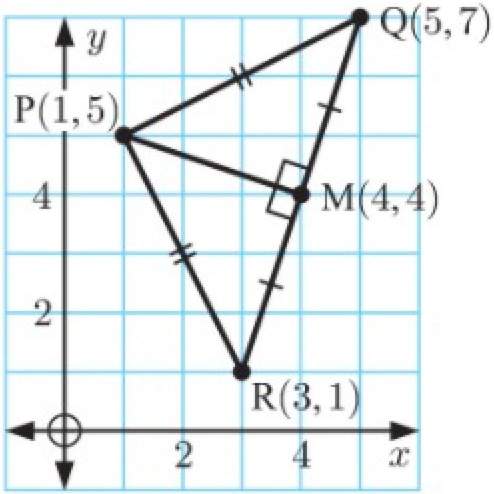
b **i** [BC] $\parallel$ [EF] **ii** [DE] $\perp$ [FA]

4 $k = -2$ **5** **a** $t = 4$ **b** $t = 4$ **c** $t = 14$ **d** $t = \frac{22}{7}$

6 **a** PQ $=$ PR $= 2\sqrt{5}$ units **b** M(4, 4)

c gradient of [PM] $= -\frac{1}{3}$,
gradient of [QR] $= 3$, and
their product is -1
$\therefore$ [PM] $\perp$ [QR]

d



7 M(0, 3) and N(3, 3)

a gradient of [MN] $= 0$, gradient of [AC] $= 0$
$\therefore$ [MN] $\parallel$ [AC]

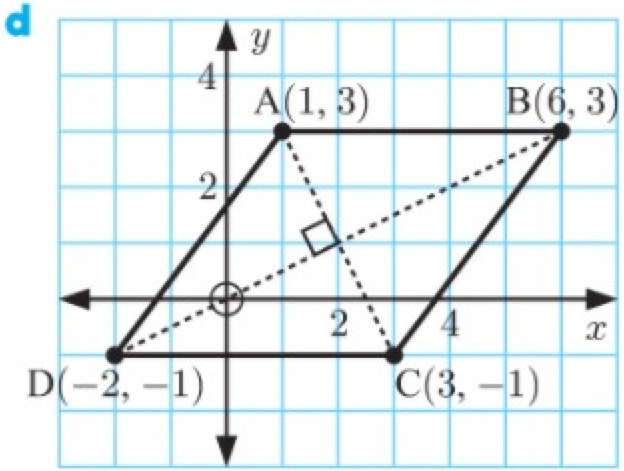
b MN $= 3$ units and AC $= 6$ units $\therefore$ MN $= \frac{1}{2}$(AC)

8 **a** All sides have length 5 units. $\therefore$ ABCD is a rhombus.

b (2, 1) and (2, 1)

c gradient of [AC] $= -2$,
gradient of [BD] $= \frac{1}{2}$
and their product $= -1$
$\therefore$ [AC] $\perp$ [BD]

d



9 **a** **i** P(0, 5) **ii** Q$(\frac{9}{2}, 2)$ **iii** R$(\frac{1}{2}, -\frac{5}{2})$ **iv** S$(-4, \frac{1}{2})$

b **i** $-\frac{2}{3}$ **ii** $\frac{9}{8}$ **iii** $-\frac{2}{3}$ **iv** $\frac{9}{8}$

c PQRS is a parallelogram.

10 **a** $s = 6$ **b** **i** $\frac{1}{2}$ **ii** -2

c gradient of [PS] $\times$ gradient of [SQ] $= -1$
$\therefore$ $\widehat{PSQ} = 90^\circ$

EXERCISE 6D.2

1 **a** gradient of [AB] $= \frac{4}{3}$, gradient of [BC] $= \frac{5}{4}$

$\therefore$ not collinear

b gradient of [PQ] = gradient of [QR] $= \frac{6}{5}$

$\therefore$ P, Q, R are collinear

c gradient of [RS] $= -\frac{3}{11}$, gradient of [ST] $= -\frac{3}{2}$

$\therefore$ not collinear

d gradient of [AB] = gradient of [BC] = 3

$\therefore$ A, B, C are collinear

2 **a** $c = 3$ **b** $c = -5$

3 **a** M$(1, -4)$

b gradient of [AM] = gradient of [MC] = 1, $\therefore$ collinear

c gradient of [AC] = 1, gradient of [BD] = -1,

$\therefore$ perpendicular